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On the Moduli Spaces of M(atric)-Theory Compactifications

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Abstract

By identifying the moduli space of coupling constants in the SYM description of toroidal compactifications of M(atric)-Theory, we construct the M(atric) description of the moduli spaces of Type IIA string theory compactified on T^n . Addition of theta terms to the M(atric) SYM produces the shift symmetries necessary to recover the correct global structure of the moduli spaces. Up to $n = 3$, the corresponding BPS charges transform under the proper representations of the U-duality groups. For $n = 4, 5$, if we make the *ansatz* of including the BPS charges corresponding to the wrapped M-theory 5-brane, the correspondence with Type IIA continues to hold. However, for $n = 6$, we find additional charges for which there are no obvious candidates in M(atric)-Theory.

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1. Introduction

One of the principles which drove the recent “revolution” in string theory was Witten’s discovery of M-Theory as a limit of Type IIA strings. Building on results of Townsend [1] and others, he realized that the spectrum of BPS-saturated threshold bound states of RR charges in Type IIA string theory were in direct correspondence with the Kaluza-Klein spectrum of the $D = 11$ supergravity multiplet with the 11th dimension compactified on a circle of radius $r = \lambda^{2/3}$, where λ is the IIA string coupling [2]. The eleven-dimensional theory describing the Type IIA theory at strong coupling later came to be called M-Theory. Another crucial development was Polchinski’s identification of the RR charged states with D-branes [3], which provided a conformal field theoretic description of strings in the presence of RR p-branes (see [4,5] for discussions and more recent progress).

Witten soon showed that, for N coincident D-branes, the $U(1)^N$ RR gauge group is enhanced to $U(N)$. The extra gauge bosons arise as the lowest modes of the open strings which stretch between the branes [6]. The low energy effective action one obtains is given by the dimensional reduction of $\mathcal{N} = 1$, $D = 9 + 1$ SYM to the D-brane world-volume. In this picture, separating coincident D-branes is equivalent to breaking part of the $U(N)$ via the Higgs mechanism in the SYM theory. At very strong coupling, the D-brane states are lighter than any massive excitation of the string, and their dynamics decouples at low energies. As shown by Danielsson, Ferretti, and Sundborg [7] and Kabat and Pouliot [8], the truncation of the spectrum to the lowest lying states reproduces the correct gravitational interactions of these branes, crucial to their interpretation as Kaluza-Klein supergravitons.

M(atr ix)-Theory took shape when Banks, Fischler, Shenker, and Susskind [9] realized that all massive string excitations as well as antibranes (with negative p_{11}) decouple in the infinite momentum frame in the 11-direction. The D0-branes are interpreted as partons and the dynamics of N partons is exactly described by $U(N)$ supersymmetric quantum mechanics. To recover eleven-dimensional physics in the infinite momentum frame, one takes the limit $N, r \rightarrow \infty$, $N/r \rightarrow \infty$, where r denotes the radius of the 11-dimension. M(atr ix)-Theory, if it is correct, provides the first non-perturbative formulation of M-Theory.

Toroidal compactifications of M(atr ix)-Theory were investigated by Taylor [10], where the effective action for compactification on T^d was shown to be $d + 1$ -dimensional SYM theory with the dual torus \tilde{T}^d as its base space. Several aspects of wrapped membranes and T and U-duality in these M(atr ix)-Theory compactifications are discussed in [11,12, 13,14,15]. The states corresponding to different wrapped membranes can be interpreted in terms of the topological quantum numbers of time-independent classical solutions of the equations of motion. In particular, the first Chern class (magnetic flux) corresponds to

wrapped D2-branes, the second Chern class (instanton number) corresponds to wrapped longitudinal 5-branes [16] (D4-branes in the IIA theory [12,17]), and the third Chern class was conjectured [18] to be the correct description for the wrapped D6-brane [19] of the IIA theory.

An explicit construction of the moduli space of scalars for M(atrix)-Theory compactifications is of great importance in evaluating its validity and in making further progress in understanding the larger role of M-Theory. For toroidal compactifications beyond the 3-torus, the M(atrix) SYM theory is non-renormalizable [18,14,20,21], at least by power-counting. In this paper, we focus on properties which are independent of the particular dynamics of the theories in question, namely the moduli space and BPS spectrum. We provide a construction of the moduli spaces of M(atrix)-Theory compactified on tori. For spacetime dimensions $D = 7, \dots, 10$, we have agreement with the moduli spaces and central charges of the type IIA string theory. For the simplest cases, we give the explicit map between the M(atrix) and SYM moduli. For $D = 6$ and below, the central charge corresponding to the wrapped transverse 5-brane is missing from the M(atrix)-Theory description. If we make the *ansatz* of including the 5-brane charges by hand, the central charges do assemble into the correct representations of the U-duality group. For $D = 5$ and below, there are moduli which are missing from the M(atrix) description. Finally, in $D = 4$, there are new central charges in IIA string theory and in M-Theory for which there no plausible candidates in M(atrix)-Theory.

2. Toroidal Compactifications of Type IIA String Theory

As a first step in constructing the moduli space of M(atrix)-Theory compactifications on general manifolds, it is instructive to reproduce the moduli space of toroidal compactifications of weakly coupled Type IIA string theory and review the standard arguments for U-duality on the central charge of these moduli.

From the conformal field theory point of view, the most general toroidal compactification of the Type IIA theory to $10 - d$ dimensions is given by an even self-dual Lorentzian lattice $\Lambda^{d,d}$, which corresponds to left-movers and right-movers living on different tori [22]. After a Poisson resummation, this is seen to be equivalent to giving the metric, $g_{\mu\nu}$, and the antisymmetric tensor field, $B_{\mu\nu}$, constant background values on the torus [23].

For simplicity, consider T^2 . It is instructive to think of the compactification as occurring in two stages. In the first stage, compactification of X_1 on a circle, we generate two extra

$U(1)$ gauge fields which correspond to the Kaluza-Klein reduction of the metric

$$g_{\mu\nu} \rightarrow g_{\mu\nu} \oplus g_{\mu 1} \oplus g_{11} \quad (2.1)$$

and the antisymmetric tensor field

$$B_{\mu\nu} \rightarrow B_{\mu\nu} \oplus B_{\mu 1}. \quad (2.2)$$

The parameter g_{11} controls the size of the circle. On the other hand, $g_{\mu 1}$ and $B_{\mu 1}$ are $U(1)$ gauge fields in the 9-dimensional theory. On further compactification to eight dimensions, g_{22} will control the size of the second circle. The third component of the metric on T^2 , g_{12} , can be viewed as a Wilson line for the corresponding $U(1)$ field. In a similar fashion, B_{12} can be viewed as a Wilson line for the $U(1)$ gauge field arising from $B_{\mu\nu}$. The net effect is that string states which are charged under both $U(1)$ s in the 1 or 2 dimensions correspond to the left and right moving modes of the string in these directions. They are therefore described in terms of their momentum and winding around the torus. The asymmetry between left-movers and right-movers is evident because the Wilson lines generate shifts in the quantization of momenta between the 1 and 2 directions which are proportional to the windings.

So far, the values of A_μ and $A_{\mu\nu\rho}$, which come from the Ramond-Ramond sector and also give $U(1)$ gauge fields in lower dimensions, have been totally ignored. The elementary string doesn't carry RR charge, so the description of the moduli space is greatly simplified. This is somewhat fortunate, since it is not possible to describe general RR backgrounds in terms of a worldsheet SCFT. However, as one takes the theory to strong coupling, RR charged solitons of the string become light, so that the effect of the RR gauge fields can no longer be ignored. At weak string coupling, these RR charged states correspond to D-branes [3], which do admit a conformal field theoretic description. When one compactifies enough dimensions, D p -branes can wrap around p -cycles and give new point-like excitations in the low energy spectrum. Wilson lines for these RR gauge potentials generate additional shifts in the quantization of momenta along compact directions. Moreover, invariance under large gauge transformations makes the configuration space of these gauge potentials compact. Under U-duality, all these states mix, therefore the full symmetry of the string theory vacua is enlarged with respect to the Narain compactifications.

The RR moduli on T^n include n Wilson lines of the RR 1-form and $n(n-1)(n-2)/3!$ periods of the RR 3-form. We can therefore express the dimension of the moduli space for

compactification on T^n , for $n \leq 4$, as

$$\begin{aligned} \dim \mathcal{M}_{\text{IIA}[T^n]} = & 1 \text{ dilaton VEV} + n^2 \text{ Narain moduli} + n \text{ Wilson lines of the RR 1-form} \\ & + \frac{n(n-1)(n-2)}{3!} \text{ periods of the RR 3-form.} \end{aligned} \quad (2.3)$$

For larger n , there are additional scalars obtained by dualizing the 1 and 3-forms. For example, in five dimensions $A_{\mu\nu\lambda}$ dualizes into a scalar. Similarly, for $n \geq 7$, there are moduli generated by dualizing the 1-form A_μ into a scalar. Therefore, when $n > 5$ there are, in addition to the moduli counted in (2.3),

$$\begin{aligned} & \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \text{ “duals” of the RR 3-form} \\ & + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{7!} \text{ “duals” of the RR 1-form.} \end{aligned} \quad (2.4)$$

The local structure of the moduli space is dictated by low-energy supergravity considerations to be one of the homogeneous spaces listed in Table 1. The true moduli space is obtained by modding out by the U-duality group [24]. These U-dualities act on the central charge of the theory. Let us recall how U-duality is realized in the string and M-Theory pictures, following [24,2]. We will explicitly consider the cases of eight and four spacetime dimensions, then recall the group disintegration properties [25] to summarize the results in intermediate dimensions.

First we consider type IIA string theory on a 2-torus. Central charges couple to the $U(1)$ vectors obtained by saturating all but one index of a p -form. From the NS-NS sector, there are four such $U(1)$ s, two from $g_{\mu a}$ and another two from $B_{\mu a}$, collectively forming the vector **4** of the T-duality group, $SO(2, 2, \mathbb{Z})$. In the RR sector, there are two more $U(1)$ s, A_μ and $A_{\mu 12}$, transforming as the spinor **2** of $SO(2, 2, \mathbb{Z})$. Together these form the **(3, 2)** representation of the U-duality group $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$. In the dual picture of M-Theory on a 3-torus, the gauge fields in the **(3, 2)** are the three “electric” gauge fields which arise from the metric, $g_{\mu a}$, and the 3 “magnetic” gauge fields which arise from the 3-form, $A_{\mu ab}$.

In four spacetime dimensions, the IIA theory has 12 $U(1)$ s in the NS-NS sector, with 24 corresponding electric and magnetic charges, while in the RR sector there are 16 $U(1)$ s, with 32 corresponding electric and magnetic charges. Collectively, the charges form the **56** of the U-duality group, $E_{7(7)}$. For M[T^7], there are 7 $g_{\mu a}$ and 21 $A_{\mu ab}$, with the electric and magnetic charges coupling to these again generating the **56** of $E_{7(7)}$. The classification of the objects which carry these charges in the IIA and M-Theory pictures is given in Table 2.

In higher spacetime dimensions, the structure of the U-duality groups can be understood from the decomposition of the **56** of $E_{7(7)}$. The sequence of disintegrations is given in Table 1.

D	Moduli Space	U-duality Group, Γ	Rep. of the Central Charge under Γ
4	$\Gamma \backslash \frac{E_{7(7)}}{SU(8)}$	$E_{7(7)}(\mathbb{Z})$	56
5	$\Gamma \backslash \frac{E_{6(6)}}{Sp(4)}$	$E_{6(6)}(\mathbb{Z})$	$1 + \mathbf{27} \quad 1 + \tilde{27}$
6	$\Gamma \backslash \frac{SO(5,5)}{SO(5) \times SO(5)}$	$SO(5,5, \mathbb{Z})$	$1 + 10 + \mathbf{16} \quad 1 + 10 + \tilde{16}$
7	$\Gamma \backslash \frac{SL(5)}{SO(5)}$	$SL(5, \mathbb{Z})$	$1 + 5 + \mathbf{10} \quad 1 + \tilde{5} + 10$
8	$\Gamma \backslash \left(\frac{SL(3)}{SO(3)} \times \frac{SL(2)}{SO(2)} \right)$	$SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$	$(1, 1) + (\tilde{3}, 1) + (\mathbf{3}, \mathbf{2}) \quad (1, 1) + (3, 1) + (\tilde{3}, 2)$
9	$\Gamma \backslash \frac{GL(2)}{SO(2)}$	$SL(2, \mathbb{Z}) \times \mathbb{Z}_2$	$1 + 2 + \mathbf{1} + \mathbf{2} \quad 1 + \tilde{2} + 1 + \tilde{2}$

Table 1: U-duality and central charges. The central charge in D spacetime dimensions belongs to the bold representation of the U-duality group and the arrows indicate how the representation decomposes under the U-duality group in $D + 1$ dimensions.

3. The Moduli Spaces of M(atrix)-Theory on Tori

M(atrix)-Theory allegedly contains $D = 11$ supergravity in its low energy spectrum, and moreover describes all the physics relevant to the strong coupling limit of Type IIA string theory. Therefore, the moduli space of string vacua should arise naturally in the language of M(atrix)-Theory. In this section, we will explicitly compute these moduli spaces to verify this claim.

In discussing M(atrix)-Theory, we consider the case in which the spatial dimension defining the infinite momentum frame is non-compact, *i.e.*, we consider the full $N, r_{\text{IMF}} \rightarrow \infty$, $N/r_{\text{IMF}} \rightarrow \infty$ limit. Therefore, when comparing a toroidal compactification with the IIA theory, we do not compactify the infinite momentum frame and one of the radii of the torus will correspond to the IIA dilaton. Finally, in light of the fact that the SYM theory that describes compactified M(atrix)-Theory is formulated on the dual torus, we employ a notation in which the duality relationships are

$$\text{IIA}[T^{d-1}] \sim \text{M}[T^d] \sim \text{M(atrix)}[\tilde{T}^d]. \quad (3.1)$$

M(atrix)-Theory on a torus \tilde{T}^d is precisely SYM on $\tilde{T}^d \times \mathbb{R}$ [10]. Part of the moduli space is described by the moduli of constant metrics of unit volume on the d -torus modulo

IIA [T^6]	M [T^7]	M(atrix) [\widetilde{T}^7]
1 D0-brane charge 6 momentum modes	7 momentum modes	7 electric fluxes
6 winding modes 15 wrapped D2-branes	21 wrapped 2-branes	21 magnetic fluxes
6 wrapped NS-NS 5-branes 15 wrapped D4-branes	21 wrapped 5-branes	?
1 wrapped D6-brane 6 KK monopoles	7 KK monopoles	??

Table 2: The $U(1)$ charge-carrying pointlike states in four spacetime dimensions in the IIA [T^6], M [T^7], and M(atrix) [\widetilde{T}^7] pictures.

diffeomorphisms. The space of such metrics is $SL(d, \mathbb{Z}) \backslash SL(d, \mathbb{R}) / SO(d)$. Note – and this is very important – that $SL(d, \mathbb{Z})$ is not a symmetry of the Yang-Mills theory on \widetilde{T}^d . However, we shall see that $SL(d, \mathbb{Z})$ is a symmetry of the spectrum of BPS states and, very likely, of their interactions¹.

In addition, there are two more *dimensionful* parameters: the size of the torus, $V = \text{Vol}(\widetilde{T}^d)$, and the coupling constant, g , with dimension $[g] \sim (\text{mass})^{(3-d)/2}$. One combination of these sets the mass scale for the spacetime theory. The other combination, the dimensionless coupling constant

$$\tilde{g} = gV^{(d-3)/2d}, \quad (3.2)$$

is a modulus of the theory (see also the discussion in [21]).

In describing the moduli space, we must also give an exact description of how the momentum labels are found for the compact directions, as well as an explicit construction of the deformations of the field theory that give rise to the shifts in momenta found in the IIA theory. We will now show explicitly how the quantization of momenta in the compact directions is obtained.

First, notice that, as $H_1(\widetilde{T}^d, \mathbb{Z}) = \mathbb{Z}^d$, one can have Wilson lines for the $U(1)$ gauge fields along each of these cycles. These correspond to the values of the zero-modes of the gauge fields. As the SYM theory is gauge invariant, the zero-modes, A_μ^0 , satisfy $A_\mu^0 dx^\mu \in H^1(\widetilde{T}^d, \mathbb{R})$.

¹To lowest order, the interactions between the BPS states arise from $F_{\mu\nu}^4$ terms. The resulting scattering amplitudes are proportional to the square of a bilinear in *differences* of BPS charges. The bilinear is formed by dotting the differences of BPS charges into the same quadratic form as appears in the BPS mass formula. The resulting formula is naturally U-invariant.

However, since the zero-modes are only well defined modulo the shifts generated by large gauge transformations, we really should refine this statement to

$$A_\mu^0 dx^\mu \in H_1(\tilde{T}^d, \mathbb{R})/\mathbb{Z}^d. \quad (3.3)$$

Hence the gauge field configuration space of Wilson lines is a torus that has the same shape as T^d . Conjugate momenta will be proportional to $\dot{A}_\mu dx^\mu = F_{0\mu}$, where we have taken $A_0 = 0$, and are quantized in units of the dual torus \tilde{T}^d . Therefore the electric fluxes provide the correct description of the momenta in the compact directions.

In IIA compactifications, Wilson lines produce shifts in momenta which are proportional to the winding numbers of the string. Here in the M(atrix) SYM theory, the electric flux is also allowed shifts in quantization. These shifts are generated by the addition of a total time derivative that does not affect the equations of motion, but that does affect the quantization of momenta. This will be our approach for M(atrix)-Theory. We will add total time derivatives to the action and we will interpret each one of these terms as an allowable deformation. As we will see later, counting all these terms properly will give the coordinates of the IIA moduli space. They will also have all of the shift symmetries of the corresponding moduli, which in the string theory correspond to large gauge transformations that generate shifts of integral multiples of $2\pi i$ in the world-volume action of membranes wrapped around non-trivial cycles.

3.1. M(atrix)-Theory in Ten Dimensions

One phenomenon that we will meet as we consider SYM theory in various dimensions is the possibility of adding topological terms to the action, that is, the integrals of various characteristic classes, $\mathcal{P}(V)$, of the vector bundle V . Not all of these will lead to sensible physics. We need to require that widely separated clusters of D0-branes should approximately decouple. In the M(atrix)-Theory language, this means that when the vector bundle V is a direct sum, the action should factorize. For the topological terms, then, we need to restrict ourselves to characteristic classes which satisfy

$$\mathcal{P}(V_1 \oplus V_2) = \mathcal{P}(V_1) + \mathcal{P}(V_2). \quad (3.4)$$

Furthermore, we wish to respect the charge-conjugation symmetry² which, in the M(atrix) theory language, exchanges the vector bundle V with its dual,

$$\mathcal{P}(V) = \mathcal{P}(V^*). \quad (3.5)$$

²In M-Theory, this is CP , where $C : A \rightarrow -A$ and $P : x^i \rightarrow -x^i$, for $i = 1, \dots, 9$.

For $U(N)$ bundles, the first restriction (3.4) means that we should restrict ourselves to considering the Chern character, $ch(V)$. The second restriction (3.5) means that we should consider only the even³ Chern characters,

$$ch_2(V) = \frac{1}{8\pi^2} \text{Tr } F \wedge F, \quad ch_4(V) = \frac{1}{4!(2\pi)^4} \text{Tr } F \wedge F \wedge F \wedge F,$$

etc. These terms will only start to make their appearance once we have compactified a sufficient number of dimensions.

The counting of moduli for the case of M(atrix)-Theory on S^1 is rather trivial, since there is only one modulus, namely the SYM coupling constant. This corresponds to the VEV of the dilaton in the Type IIA string theory and we can use (3.2) to obtain this correspondence. We compare the value of the D0-brane tension,

$$T_{D0} = M_s e^{-\phi^{(10)}}, \quad (3.6)$$

where $\phi^{(10)}$ is the 10-dimensional dilaton, with the M(atrix) value $T_{D0} = 1/r$. Noting that the eleven-dimensional Planck mass is related to the string scale via $M_P = e^{-2\phi^{(10)}/9} M_s$, we find the expected relationship [2]

$$r = M_P^{-1} e^{2\phi^{(10)}/3}. \quad (3.7)$$

3.2. M(atrix)-Theory in Nine Dimensions

Now consider M(atrix)[\tilde{T}^2], whose moduli are described by the dimensionless coupling constant and the 2 parameters of a unit volume metric on the 2-torus. Equivalently, instead of the volume-one metric, we can consider the complex structure of the torus, which encodes the same information. Therefore, the dimension of the moduli space is equal to that for IIA[S^1] as obtained from (2.3). In fact, the global structure is the same in both cases. The complex structure, $\tau_{\text{cplx.}}$, lies in the fundamental domain \mathcal{F} in the upper half plane, while the coupling constant is a positive real number, so

$$\mathcal{M}_{\text{M(atrix)}[\tilde{T}^2]} = \mathcal{F} \times \mathbb{R}^+. \quad (3.8)$$

We can arrange the moduli as

$$\begin{aligned} (w + ie^{-\phi}, M_s^{-1} R) & \quad \text{for IIA}[S^1] \\ (\tilde{\varphi} + ie^{-\phi}, 2M_s R^{-1}) & \quad \text{for IIB}[S^1] \\ (\tau_{\text{cplx.}}, 1/\tilde{g}^2) & \quad \text{for M(atrix)}[\tilde{T}^2], \end{aligned} \quad (3.9)$$

³The first Chern character, $ch_1(V) = c_1(V) = \frac{1}{2\pi} \int \text{Tr } F$, actually has a direct connection to the Galilean invariance of the theory. The theta angles for this term are, in fact, the angles between the sides of the torus and the light-cone direction. Since this term only couples to the $U(1)$ component of the $U(N)$ gauge group, this term is decoupled from the interactions, therefore providing a crucial test of the Galilean invariance.

where $w = \oint A \cdot dx$ is the RR Wilson line of the IIA theory, $\tilde{\varphi}$ is the RR scalar of the IIB theory, ϕ is the 9-dimensional dilaton, and R is the radius of the circle in the IIA compactification. In each case, the moduli space is

$$SL(2, \mathbb{Z}) \times \mathbb{Z}_2 \setminus \frac{GL(2, \mathbb{R})}{SO(2)} = \mathcal{F} \times \mathbb{R}^+, \quad (3.10)$$

so that the M(atrrix)-Theory moduli space exactly agrees with that of the string theories. The equations (3.9) give the mapping between the parameterizations and we find that (3.7) is satisfied. Additionally, we see, heuristically, the correspondence between the $\tilde{g} \rightarrow \infty$ limit of the SYM theory and the Type IIB theory in ten dimensions.

The $SL(2, \mathbb{Z})$ component of the U-duality group in this picture acts on the two electric fluxes and one magnetic flux, which form the $\mathbf{2} + \mathbf{1}$ representation. This is not a symmetry of the 2+1-dimensional SYM action, but is rather a symmetry of the BPS spectrum.

3.3. M(atrrix)-Theory in Eight Dimensions

M(atrrix)[\tilde{T}^3] has 6 moduli corresponding to the coupling constant and the metric on the torus \tilde{T}^3 . The question now is, what corresponds to the VEV of B_{12} in M(atrrix)-Theory? The effect of this VEV in string theory is to shift the quantization of momenta in the partition function of wrapped strings along the 2-cycle dual to B . In M-Theory, these strings correspond to 2-branes which are wrapped around the whole of T^3 . In M(atrrix)-Theory these wrapped membranes are represented by states that carry magnetic flux in, say, the 1,2-direction with momentum along the 3-direction. In the string picture, this corresponds to winding in the 2-direction with momentum along the 3-direction. Exchange of winding and momentum is consistent with a permutation of the 2 and 3-directions in M(atrrix)-Theory. Note that, if momentum is carried along the 1-direction, from the IIA point of view, this state is a soliton, so it corresponds to a wrapped D2-brane.

For these wrapped string states, the shift in momentum requires states that are both wrapped and which carry momentum. As a result, in the partition function at fixed momentum and winding, we will have an action proportional to

$$v^2 + w^2 + v \cdot b \cdot w, \quad (3.11)$$

where v corresponds to the classical velocity around the cycle, w is the winding, and b is the expectation value of $B_{\mu\nu}$. Since w is held fixed and this is a total time derivative, we obtain the expected shift in the quantization of the momenta

$$\Delta p \sim b \cdot w. \quad (3.12)$$

Finally, as we are in 3+1 dimensions, we have our first opportunity to add a topological term to the action. We add

$$2\pi b \int ch_2(V) = \frac{b}{4\pi} \int \text{Tr } F \wedge F. \quad (3.13)$$

As $2\pi b$ is an angle, $b \sim b+1$, and combines with the Yang Mills coupling to form the complex gauge coupling $b + \frac{4\pi i}{g^2}$ of the $\mathcal{N} = 4$, $D = 4$ SYM. The shift (3.12) in the momentum is, in the context of M(atrrix)-Theory, simply a manifestation of the Witten effect [26], whereby the electric charge of a state is shifted by a term proportional to θ times the magnetic charge.

Now the T-duality of M(atrrix)[\tilde{T}^3] was studied in [11,12], where it was noted that, since the SYM is conformal, the $SL(2, \mathbb{Z})$ part of the U-duality group is insured by S-duality⁴. In this case, the $SL(2, \mathbb{Z})$ S-duality acts on the complex coupling constant $b + \frac{4\pi i}{g^2}$ and there is a $SL(3, \mathbb{Z})$ action on the 3 electric fluxes and 3 magnetic fluxes, which form the $(\mathbf{3}, \mathbf{2})$ representation of the combined U-duality group $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$. Even though the SYM theory is conformal, the $SL(3, \mathbb{Z})$ component of the U-duality group is not a symmetry of the action. It is only a symmetry of the BPS spectrum.

We would like to see the explicit correspondence between the SYM moduli and those of the IIA picture. The 7 IIA[T^2] moduli are, from our discussion in section 2, given by the dilaton, ϕ , four Narain moduli (the complex structure, $\tau = \tau_1 + i\tau_2$, and $\rho = B + ig$), as well as the two RR Wilson lines, $w_a = \oint A_a dx^a$. The T-duality group is $SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z})$, where the first factor acts on τ and the second on ρ . This gets promoted to the full U-duality group, $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$. We would like to see how the five additional moduli combine with τ to form the homogeneous space $SL(3, \mathbb{R})/SO(3)$.

To this end, we consider the action of the $SL(2, \mathbb{Z}) \subset SL(3, \mathbb{Z})$ subgroup, $\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}$, which acts as $\tau \rightarrow \frac{a\tau+b}{c\tau+d}$ on the complex structure of the torus⁵. It will prove useful to employ the GKD decomposition of an $SL(n, \mathbb{R})$ matrix, M , into the product

$$M = U D \mathcal{O}, \quad (3.14)$$

where U is upper-triangular (with 1s on the diagonal), D is diagonal with $\det D = 1$, and \mathcal{O} is orthogonal. This decomposition is the natural one to use to parameterize the quotient space $SL(3, \mathbb{R})/SO(3)$. To find the dependence on τ_1 and the w_a , we consider the left action of the Borel subgroup, $\begin{pmatrix} 1 & n & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ of $SL(2, \mathbb{Z})$ on M . This affects only the matrix U in the above decomposition. Acting with $S = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, the result can be brought back into the

⁴It is of incredibly good fortune that M(atrrix)-Theory is based on $U(N)$, a self-dual group. For a group which is not self-dual, only a subgroup of $SL(2, \mathbb{Z})$ preserves the gauge group under S-duality [27].

⁵The $SL(2, \mathbb{Z})$ does not act faithfully on τ , but it does act faithfully on the Wilson lines, $w_{1,2}$.

form (3.14) by the right action of an $SO(3)$ matrix. This determines the dependence on τ_2 . Finally, the dilaton dependence is obtained by analyzing the mass spectrum obtained from the quadratic form on the $(\mathbf{3}, \mathbf{2})$ representation,

$$MM^T \otimes \frac{1}{\text{Im } \rho} \begin{pmatrix} |\rho|^2 & \rho_1 \\ \rho_1 & 1 \end{pmatrix} (M_{pl}^{(8)})^2, \quad (3.15)$$

and comparing with the string result (3.6). We find that $SL(3, \mathbb{R})/SO(3)$ is parameterized by

$$M = \begin{pmatrix} 1 & \tau_1 & w_2 \\ 0 & 1 & w_1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\tau_2} e^{\phi/3} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\tau_2}} e^{\phi/3} & 0 \\ 0 & 0 & e^{-2\phi/3} \end{pmatrix}, \quad (3.16)$$

where now ϕ is the eight-dimensional dilaton. From the IIA string point of view, this is rather surprising, since the theory on the 2-torus actually possesses the symmetry of an eleven-dimensional theory on the 3-torus. A metric of unit volume on the 3-torus is determined from $g_{T^3} = MM^T = UD^2U^T$. Therefore the radius of the eleventh dimension, (3.7), is manifest in this treatment. With hindsight, we, of course, realize this as the manifestation of M-Theory [1,2].

To summarize, the complex coupling constant of the SYM theory, $b + \frac{4\pi i}{g^2}$, directly maps to the IIA modulus ρ . The mapping between the $SL(3, \mathbb{R})/SO(3)$ M(atrix) SYM moduli, namely the components of the metric on the 3-torus, and the corresponding IIA moduli is obtained by comparing (3.16) with the metric of unit volume on the 3-torus, $g = MM^T$, where

$$M = \begin{pmatrix} 1 & \frac{g_{12}g_{33} - g_{13}g_{23}}{g_{22}g_{33} - g_{23}^2} & \frac{g_{13}}{g_{33}} \\ 0 & 1 & \frac{g_{23}}{g_{33}} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{g_{22}g_{33} - g_{23}^2}} & 0 & 0 \\ 0 & \sqrt{g_{22} - \frac{g_{23}^2}{g_{33}}} & 0 \\ 0 & 0 & \sqrt{g_{33}} \end{pmatrix}. \quad (3.17)$$

3.4. M(atrix)-Theory in Seven Dimensions

M(atrix)[\tilde{T}^4] has nine moduli which describe the unit volume metrics on the dual 4-torus in addition to the coupling constant. The other four parameters (which in M-Theory correspond to the different A_{ijk} cycles) are given by integrals

$$\sum_{e_{ijk}} \frac{A_{ijk}}{4\pi} \int dt \int_{e_{ijk}} \text{Tr } F \wedge F, \quad (3.18)$$

where e_{ijk} is a basis for the homology 3-cycles of T^4 . There are four such 3-cycles. Again, the quantities $2\pi A_{ijk}$ are angles in the same manner as the QCD θ angle. They multiply topological invariants, and therefore don't modify the equations of motion of M(atrrix)-Theory, except at the expected shifts in the momentum quantization.

Notice that, at this point, we are dealing with a 5-dimensional gauge field theory and the renormalizability of the theory is doubtful. In four dimensions, this theory is finite to all orders in perturbation theory, and the low energy effective action receives no perturbative corrections. In [18], the 1-loop contribution to the β -function was studied in these SYM theories. In 5, 6, and 7 dimensions, one could explicitly show that the theory does not get renormalized at 1-loop. However, in 8 dimensions a logarithmic divergence was found in $F_{\mu\nu}^4$ at 1-loop, so that the perturbation theory breaks down. This divergence was related to the IR divergence present in gravity in four spacetime dimensions due to massless particle exchange, but, in general, these divergences should convince one to take perturbative calculations with a grain of salt.

Beyond one loop, there is little explicitly known. However, some insight can be gained by considering the heterotic string compactified on a torus. In that theory⁶, there are no perturbative corrections (finite or infinite) to $F_{\mu\nu}^2$. There are, of course, finite corrections to $F_{\mu\nu}^4$ and higher terms. If we view the string theory as a short-distance cutoff of the (maximally-supersymmetric) SYM theory, we may expect some of these corrections to diverge as we send the cutoff away. However, since there were no finite corrections to $F_{\mu\nu}^2$, we expect it to stay unrenormalized, even as we take the cutoff away. The next dangerous term is $F_{\mu\nu}^4$, but simple power counting says that this is an irrelevant operator for dimensions less than 8. One suspects that the string theory result is a consequence of the SYM having maximal supersymmetry. If so, one might expect that the result might hold for the $U(N)$, $N \rightarrow \infty$, theories that we are actually interested in.

These observations suggest that the specific IR behavior of these gauge theories must play a significant role [14,20,21]. It would therefore be very interesting to have a detailed description of the dynamics of these theories, such as that which would be obtained by relating them to IR fixed points of other theories. Rozali [14] has made the extremely interesting observation that the 4 + 1-dimensional M(atrrix)-Theory may be related to an IR fixed point of a theory in 5 + 1 dimensions [28]. Further discussion along these lines may be found in [20,21].

Whatever the status of its renormalizability, one still expects that M(atrrix) SYM contains a consistent description of the moduli and BPS states. Moreover, all of this information is

⁶We thank Vadim Kaplunovsky for discussions on this point.

encoded in quantities that are well behaved in the IR limit. We will therefore focus our discussion on the BPS properties, which do not depend on the short-distance behaviour of the theory.

We would like to know what manifold the moduli of the SYM theory parameterize. It is obvious that the components of the metric parameterize an $SL(4, \mathbb{R})/SO(4)$ subspace, but the theta angles should enhance this. We can see how this occurs by considering the U-duality action on the central charges. Under the $SL(4, \mathbb{Z})$ group of torus deformations, the 4 electric fluxes fall into the vector $\mathbf{4}$ and the 6 magnetic fluxes compose the antisymmetric tensor $\mathbf{6}_a$. This is the natural decomposition of the antisymmetric tensor $\mathbf{10}_a$ of $SL(5, \mathbb{Z})$ under its $SL(4, \mathbb{Z})$ subgroup. Therefore these charges naturally reproduce the correct representation of the U-duality group, as indicated by Table 1.

Now the four theta angles in (3.18) form a vector, $a_i = \epsilon_{ijkl} A^{jkl}/3!$ under $SL(4, \mathbb{Z})$ transformations. Therefore the theta angles appear in the U factor of the decomposition of $SL(5, \mathbb{R})/SO(5)$ and the D factor is determined up to a function of \tilde{g} ,

$$M = UD = \begin{pmatrix} U_4 & a_i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D_4 f(\tilde{g})^{-1} & 0 \\ 0 & f(\tilde{g})^4 \end{pmatrix}, \quad (3.19)$$

where

$$U_4 = \begin{pmatrix} 1 & \frac{(g_{12}g_{44}-g_{14}g_{24})(g_{33}g_{44}-g_{34}^2)-(g_{13}g_{44}-g_{14}g_{34})(g_{23}g_{44}-g_{24}g_{34})}{(g_{22}g_{44}-g_{24}^2)(g_{33}g_{44}-g_{34}^2)-(g_{23}g_{44}-g_{24}g_{34})^2} & \frac{g_{13}g_{44}-g_{14}g_{34}}{g_{33}g_{44}-g_{34}^2} & \frac{g_{14}}{g_{44}} \\ 0 & 1 & \frac{g_{23}g_{44}-g_{24}g_{34}}{g_{33}g_{44}-g_{34}^2} & \frac{g_{24}}{g_{44}} \\ 0 & 0 & 1 & \frac{g_{34}}{g_{44}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$D_4 = \text{diag} \left(\frac{1}{\sqrt{(g_{22}g_{44}-g_{24}^2)(g_{33}g_{44}-g_{34}^2)-(g_{23}g_{44}-g_{24}g_{34})^2}}, \sqrt{\frac{(g_{22}g_{44}-g_{24}^2)(g_{33}g_{44}-g_{34}^2)-(g_{23}g_{44}-g_{24}g_{34})^2}{g_{33}g_{44}-g_{34}^2}}, \right.$$

$$\left. \sqrt{\frac{g_{33}g_{44}-g_{34}^2}{g_{44}}}, \sqrt{g_{44}} \right)$$

correspond to the decomposition using (3.14) of the metric on the 4-torus, $g_4 = U_4 D_4^2 U_4^T$. The coupling constant dependence may be determined by realizing that the quadratic form MM^T extends to a quadratic form on the antisymmetric tensor of $SL(5, \mathbb{R})$. This yields the correct BPS mass formula for the charges. With g_{ij} a metric on the 4-torus of unit volume, we find, up to an overall constant which fixes the scale of the Hamiltonian,

$$\mathcal{M}^2 \sim \tilde{g}^2 g_{ij} (n^i + A^{ikl} n_{kl}) (n^j + A^{jrs} n_{rs}) + \left(\frac{2\pi}{\tilde{g}} \right)^2 g^{ij} g^{kl} n_{ik} n_{jl}, \quad (3.20)$$

where the n^i and n_{ij} are integers, so that $f(\tilde{g}) = \tilde{g}^{4/5}/(2\pi)^{2/5}$.

3.5. M(atrix)-Theory in Six Dimensions

For M(atrix)[\tilde{T}^5], there are 25 moduli, corresponding to the dimensionless coupling constant, 14 parameters of volume one metrics on the dual torus, and 10 theta angles. This is precisely the same number of moduli as appears in the case of IIA[T^4]. We expect that it would not be too hard to use an explicit parameterization of $SO(5, 5)/SO(5) \times SO(5)$ to determine the precise mapping between the SYM and string moduli, as we have done above in the higher dimensional cases. We will not, however, attempt that here.

Let us now consider the central charges. These are composed of 5 electric fluxes and 10 magnetic fluxes transforming in the vector **5** and the antisymmetric tensor **10_a** of $SL(5, \mathbb{Z})$. The spinor **16** of $SO(5, 5, \mathbb{Z})$ decomposes under the $SL(5, \mathbb{Z})$ subgroup as $\mathbf{1} \oplus \mathbf{5} \oplus \mathbf{10}_a$. Therefore, as was also realized in [21], an additional state is required to complete the desired U-duality multiplet. From the M-Theory picture, this must correspond to a transverse 5-brane which completely wraps the 5-torus. We note that, although a charge is missing here, we do, in fact, have all of the modular parameters for this compactification. It is nevertheless very important to give an explicit construction of these transverse 5-brane states in the M(atrix) model, as they are crucial to preserving U-duality.

3.6. Compactification to Lower Dimensions

So far, we have considered compactifications for which, in the M-Theory picture, transverse 5-branes do not contribute moduli. Nevertheless, we did see how they are required to complete U-duality multiplets. We can summarize the dimensions of these M(atrix) SYM moduli spaces in the same manner that we did for the IIA string in (2.3)

$$\begin{aligned} \dim \mathcal{M}_{\text{M(atrix)}[\tilde{T}^d]} = & 1 \text{ dimensionless coupling constant} \\ & + \left(\frac{d(d+1)}{2} - 1 \right) \text{ moduli of metrics of unit volume} \\ & + \frac{d(d-1)(d-2)}{3!} \text{ theta angles.} \end{aligned} \quad (3.21)$$

Substituting $d = n + 1$, we see that these formulæ agree.

However, in spacetime dimensions $D \leq 5$, the IIA theory has the additional moduli given in (2.4). Thus the first discrepancy occurs in M(atrix)[\tilde{T}^6], where we are missing one modulus in (3.21). In the M-Theory language, one can dualize the 3-form gauge field to a 6-form, and the modulus in question corresponds to a constant expectation value for this 6-form on T^6 .

The difficulty, of course, is that this is the gauge field that couples to the 5-brane, and

we do not have an explicit construction of the transverse 5-brane in M(atrrix)-Theory. If we did, we would, perhaps, be able to understand this modulus.

Nevertheless, if we continue to make the *ansatz* of including the central charges associated to the wrapped transverse 5-branes, we obtain the correct representation of the U-duality group on the central charges for M(atrrix)[\tilde{T}^6]. The central charge corresponds to 6 electric fluxes, 15 magnetic fluxes, and 6 wrapped transverse 5-branes. Our target is the **27** of $E_{6(6)}$, which decomposes as $(\mathbf{6}, \mathbf{2}) \oplus (\mathbf{15}_a, \mathbf{1})$ under the maximal subgroup $SL(6, \mathbb{Z}) \times SL(2, \mathbb{Z})$. Our collection satisfies this structure under the $SL(6, \mathbb{Z})$ group of torus deformations, but the $SL(2, \mathbb{Z})$ symmetry is not manifest. The $SL(2, \mathbb{Z})$ symmetry mixes the electric fluxes with the 5-branes, so, given the absence of an explicit construction of the transverse 5-brane, it is not surprising that we do not see it. Nevertheless, the algebraic structure that we do see is evidence enough that the correct $E_{6(6)}$ U-duality is present.

Finally, we can consider M(atrrix)[\tilde{T}^7]. In this case, including the effects of the transverse 5-brane is *not* enough to insure the proper construction of the U-duality representation. We have explicitly 7 electric fluxes and 21 magnetic fluxes, which fall into the **7** and **21** of the manifest $SL(7, \mathbb{Z})$. These are what one expects from the decomposition of the **56** of the U-duality group $E_{7(7)}$, as $\mathbf{7} + \mathbf{21} + \widetilde{\mathbf{21}} + \widetilde{\mathbf{7}}$ under the $SL(7, \mathbb{Z})$ subgroup. Our discussion above would motivate us to add to the above central charges 21 wrapped transverse 5-branes, in the $\widetilde{\mathbf{21}}$ of $SL(7, \mathbb{Z})$. There are, however, 7 additional charges, which according to our summary in Table 2, correspond to Kaluza-Klein monopoles [29,30] in the corresponding M[T^7] picture. That is, they correspond to geometries in which the 11-dimensional spacetime is not globally $\mathbb{R}^4 \times T^7$. Rather, one has a *non-trivial* T^7 -bundle over the 2-sphere at spatial infinity. Near the origin, the fibration structure goes bad, but the total space is non-singular. A non-trivial S^1 -bundle over S^2 corresponding to the monopole of charge 1 is given by the Hopf fibration $S^3 \rightarrow S^2$. Since we have seven S^1 s, there are 7 monopole charges, which form the **7** of $SL(7, \mathbb{Z})$.

It is, to say the least, unclear how this structure is to be incorporated into the M(atrrix) theory description. Perhaps this failure is related to the logarithmic divergence and the loss of one-loop finiteness of the 7 + 1-dimensional SYM theory that we discussed in section 3.4. Another possibility is that the missing charges may be related to the 6-brane charge found in [19]. If such charges did appear in the 7+1-dimensional SYM, they would, indeed, transform as the **7** of $SL(7, \mathbb{Z})$. However, by taking one of the radii of the \tilde{T}^7 to be very small, one can argue that they should contribute a single central charge for the SYM theory on $\tilde{T}^6 \times \mathbb{R}$. But this would be a disaster, as M(atrrix)[\tilde{T}^6] already gave the correct central charges to agree with IIA[T^5]. Adding one more charge would ruin the correspondence

between M(atrix)-Theory and IIA string theory in $D = 5$.

4. Conclusions

Our analysis of the toroidal M(atrix) SYM theories yields an explicit construction of their moduli spaces. The correct global structure of the moduli spaces is evident and agrees with that of the moduli spaces of IIA compactifications. Moreover, the central charge of the M(atrix) SYM theory transforms in the manner necessary to recover the U-dualities of the corresponding string theories. We emphasize that U-duality is, in general, only a symmetry of the BPS charges and is not a symmetry of the action.

It is of particular interest to us to consider how these constructions may be extended to more complicated scenarios of compactification. In particular, it is interesting to understand how far the M(atrix) model can be used to extract information of the strong coupling limit of different string theories and to provide a full description of M-Theory. Clearly, the limitations of the M(atrix) model become apparent when one compactifies on a large number of dimensions, indeed on \tilde{T}^4 . At this point, the M(atrix)-Theory is a $4 + 1$ -dimensional SYM theory, over which we have little control from a perturbative point of view. A more complete understanding of the dynamics, such as information from IR fixed points [14,20,21], is crucial to understanding these theories. However, results in this direction [14] suggest that these theories flow to tensionless string theories. It is still not possible to apply these to yield concrete statements which would extend our results.

In addition, on higher dimensional tori, the effect of M-Theory transverse 5-branes can no longer be ignored. Recent attempts to understand these objects [31,32] do not appear to lend themselves toward a description of the corresponding SYM moduli. The treatment of Lifschytz [31] relied on repeated T-dualization of the membrane. Since the full U-duality group is never a symmetry of the SYM action, we cannot make use of this technique in the approach we have outlined in this paper. In the construction of Berkooz and Rozali [32], the 5-brane, wrapped around the 5-torus, appears as a winding mode of a scalar field around the 6th dimension of the 5+1-dimensional theory found in [14]. This 5+1-dimensional theory is essentially the same as that on the world-volume of the 5-brane and involves self-dual tensors, and is not completely understood. The theory is potentially anomalous [33,34] and the self-duality of the tensor appears to spoil the possibility of extending our approach within the SYM theories. At the time this article is being written, these questions are unanswered.

The current absence of either a simple description of the dynamics or an explicit construction of the transverse 5-brane in the M(atrix) SYM formulation is certainly an impediment

to obtaining a rich understanding of M(atrrix)-Theory. Indeed, recent work of Hashimoto and Taylor [35] suggests that the SYM formulation is too simple to describe certain configurations of tilted branes [5] and branes intersecting at angles [36]. In these cases, they found that one must resort to a non-Abelian generalization of the Born-Infeld action, such as that proposed by Tseytlin [37]. Furthermore, it is even less evident how one might incorporate the presence of Kaluza-Klein monopoles into the M(atrrix) SYM, as will certainly be necessary in $D = 4$. What impact these results might have on M(atrrix)-Theory is nevertheless an exciting area of future investigation.

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References

- [1] P. K. Townsend, “The eleven-dimensional supermembrane revisited,” *Phys. Lett.* **B350** (1995) 184–187, [hep-th/9501068](#).
- [2] E. Witten, “String theory dynamics in various dimensions,” *Nucl. Phys.* **B443** (1995) 85–126, [hep-th/9503124](#).
- [3] J. Polchinski, “Dirichlet-branes and Ramond-Ramond charges,” *Phys. Rev. Lett.* **75** (1995) 4724–4727, [hep-th/9510017](#).
- [4] J. Polchinski, S. Chaudhuri, and C. V. Johnson, “Notes on D-branes,” [hep-th/9602052](#).
- [5] J. Polchinski, “TASI Lectures on D-branes,” [hep-th/9611050](#).
- [6] E. Witten, “Bound states of strings and p-branes,” *Nucl. Phys.* **B460** (1996) 335–350, [hep-th/9510135](#).
- [7] U. H. Danielsson, G. Ferretti, and B. Sundborg, “D-particle dynamics and bound states,” *Int. J. Mod. Phys.* **A11** (1996) 5463–5478, [hep-th/9603081](#).
- [8] D. Kabat and P. Pouliot, “A comment on zerobrane quantum mechanics,” *Phys. Rev. Lett.* **77** (1996) 1004–1007, [hep-th/9603127](#).

- [9] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, “M Theory as a matrix model: A conjecture,” *Phys. Rev.* **D55** (1997) [hep-th/9610043](#).
- [10] W. Taylor, “D-brane field theory on compact spaces,” *Phys. Lett.* **B394** (1997) 283–287, [hep-th/9611042](#).
- [11] L. Susskind, “T duality in M(atr)ix theory and S duality in field theory,” [hep-th/9611164](#).
- [12] O. J. Ganor, S. Ramgoolam, and W. Taylor, “Branes, fluxes and duality in M(atr)ix-theory,” *Phys. Lett.* **B492** (1997) 191, [hep-th/9611202](#).
- [13] S. Sethi and L. Susskind, “Rotational invariance in the M(atr)ix formulation of type IIB theory,” [hep-th/9702101](#).
- [14] M. Rozali, “Matrix theory and U-duality in seven dimensions,” [hep-th/9702136](#).
- [15] R. Gopakumar, “BPS states in matrix strings,” [hep-th/9704030](#).
- [16] M. Berkooz and M. R. Douglas, “Five-branes in M(atr)ix theory,” *Phys. Lett.* **B395** (1997) 196–202, [hep-th/9610236](#).
- [17] Z. Guralnik and S. Ramgoolam, “Torons and D-brane bound states,” [hep-th/9702099](#).
- [18] D. Berenstein and R. Corrado, “M(atr)ix-theory in various dimensions,” [hep-th/9702108](#).
- [19] T. Banks, N. Seiberg, and S. Shenker, “Branes from matrices,” *Nucl. Phys.* **B490** (1997) 91, [hep-th/9612157](#).
- [20] T. Banks and N. Seiberg, “Strings from matrices,” [hep-th/9702187](#).
- [21] W. Fischler, E. Halyo, A. Rajaraman, and L. Susskind, “The incredible shrinking torus,” [hep-th/9703102](#).
- [22] K. S. Narain, “New heterotic string theories in uncompactified dimensions < 10 ,” *Phys. Lett.* **169B** (1986) 41.
- [23] K. S. Narain, M. H. Sarmadi, and E. Witten, “A note on toroidal compactification of heterotic string theory,” *Nucl. Phys.* **B279** (1987) 369.
- [24] C. M. Hull and P. K. Townsend, “Unity of superstring dualities,” *Nucl. Phys.* **B438** (1995) 109–137, [hep-th/9410167](#).
- [25] B. Julia, “Group Disintegrations,” in *Superspace and Supergravity* (M. Rocek and S. Hawking, eds.), p. 331, Cambridge University Press, 1981.
- [26] E. Witten, “Theta vacua in two-dimensional quantum chromodynamics,” *Nuovo Cim.* **51A** (1979) 325.

- [27] C. Vafa and E. Witten, “A strong coupling test of S-duality,” *Nucl. Phys.* **B431** (1994) 3–77, [hep-th/9408074](#).
- [28] N. Seiberg, “Non-trivial fixed points of the renormalization group in six dimensions,” *Phys. Lett.* **B390** (1997) 169–171, [hep-th/9609161](#).
- [29] R. D. Sorkin, “Kaluza-Klein monopole,” *Phys. Rev. Lett.* **51** (1983) 87.
- [30] D. J. Gross and M. J. Perry, “Magnetic monopoles in Kaluza-Klein theories,” *Nucl. Phys.* **B226** (1983) 29.
- [31] G. Lifschytz, “A note on the transverse five-brane in M(atrix) theory,” [hep-th/9703201](#).
- [32] M. Berkooz and M. Rozali, “On transverse fivebranes in matrix theory on T^5 .” To appear.
- [33] E. Witten, “On flux quantization in M-theory and the effective action,” [hep-th/9609122](#).
- [34] E. Witten, “Five-brane effective action in M-theory,” *J. Geom. Phys.* **22** (1997) 103, [hep-th/9610234](#).
- [35] A. Hashimoto and W. Taylor, “Fluctuation spectra of tilted and intersecting D-branes from the Born-Infeld action,” [hep-th/9703217](#).
- [36] M. Berkooz, M. R. Douglas, and R. G. Leigh, “Branes intersecting at angles,” *Nucl. Phys.* **B480** (1996) 265–278, [hep-th/9606139](#).
- [37] A. A. Tseytlin, “On non-abelian generalization of Born-Infeld action in string theory,” [hep-th/9701125](#).